Code Assessment for Adaptive Mesh Refinement Simulations

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We investigated and analyzed the grid convergence issues for adaptive mesh refinement (AMR) code. These issues are related to the RAGE code and have been raised in a previous report, *"Two-dimensional convergence study for problems with exact solution: uniform and adaptive grids"* (Li et al.[1], LA-UR-05-7985).

Code verification is extremely important for science-based prediction and simulations. Previous verification focused on the convergence behavior of uniform grid. Using AMR, we can obtain more accurate results with substantially less computational cost. Ideally, AMR should achieve the same accuracy in refinement region as the corresponding fine uniform grid. We expect the results of an AMR grid at least should be better than the results of the coarse uniform grid without local refinement.

Using two different AMR packages, we have investigated three model problems that have exact solutions and represent a variety of problems. Several issues with respect to the RAGE AMR have been found: *(a) it has large initialization errors; (b) the numerical error with AMR is larger than without AMR for a high-resolution grid; (c) AMR with more than one-level refinement has larger numerical errors than with only one-level refinement, and many other issues.*

We have investigated these issues in more detail and proposed several methods to solve them. In particular, we tested a new mesh initialization for AMR solutions, and several refinement criteria to achieve the expected accuracy and convergence rate for AMR simulations.

AMR Initialization Issue: After careful examination of the initial error and RAGE AMR implementation, we have found that the RAGE AMR never generates the initial AMR mesh based on a user-input routine for exact initial conditions. The initial error is purely an interpolation error from the coarse grid to the AMR fine grid. We proposed a new mesh generation procedure for RAGE AMR to use the user-input routine. This new procedure does not require too much extra coding and eliminates the source of the initial errors (see Figure 1).

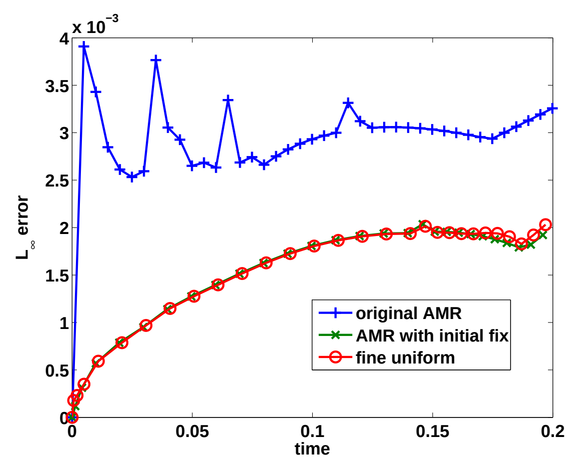

Figure 1: Numerical error (L_∞) before and after AMR initialization fix for wave's problem

We also proposed a new AMR flagging approach to turn off the activity test so that AMR does not refine everywhere for Noh's problem (see Figure 2). This approach also fixes the shock instability for Noh's 3D spherical problem (see Figure 3).

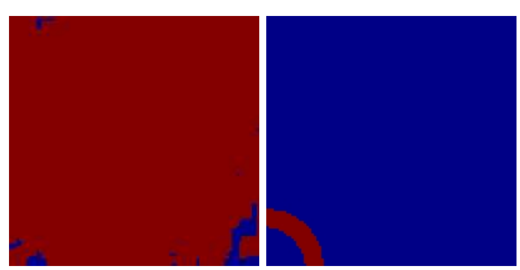

Figure 2: Mesh refinement before (left) and after (right) AMR fix for Noh's problem. The red color denotes the second level.

AMR Refinement Criteria: After detailed anal-

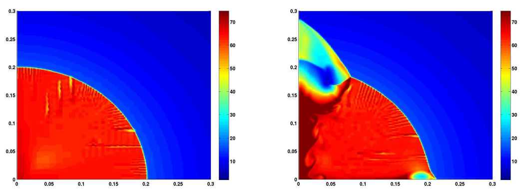

Figure 3: Fixing AMR refinement also remove the shock instability for Noh's 3D problem. Right: before fix; left: after fix.

ysis, we have found that the numerical solution at the coarse-fine interface between different levels of grid converges only in the first-order accuracy. Therefore, the error near the coarse-fine interface can quickly dominate the error in other regions if the coarse-fine interface is active and not covered by the fine grid.

We implemented and compared several refinement criteria (RC): solution gradient-based, solution curvature-based, RC of FLASH code, Richardson extrapolation type, operator recovery error source detector (ORESD) of Lapenta [3], etc. Some of them can catch the large-error region near the coarse-fine interface and refine them with the fine grid. We found the Richardson extrapolation approach and modified ORESD approach perform better than others. Several issues in RAGE AMR are solved by using the new refinement criteria.

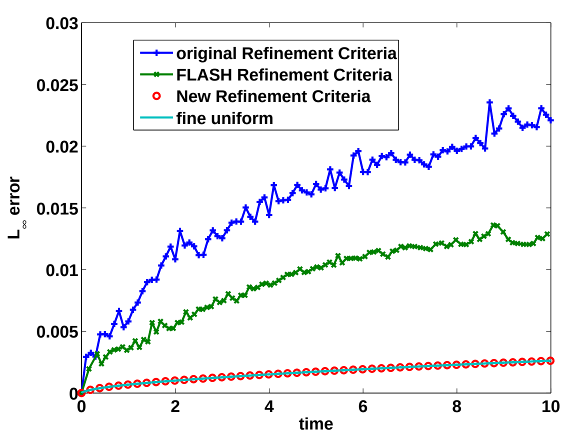

Figure 4: Numerical error for different refinement criteria.

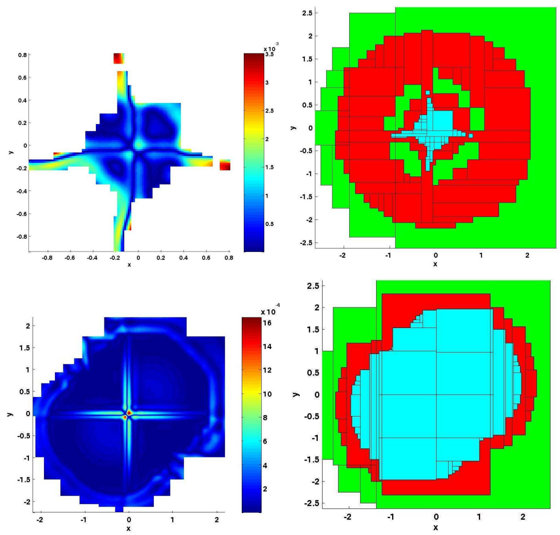

Figure 5: Numerical error and mesh refinement for AMR with 3-level refinement. The top two is for FLASH's refinement criteria and the bottom two is for ORESD.

The numerical results show that the refinement criteria play an important role in convergence behavior of AMR solutions.

Acknowledgements

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References

[1] S. Li, W. J. Rider, and M. J. Shashkov, Two-Dimensional Convergence Study for Problems with Exact Solution: Uniform and Adaptive Grids, Technical Report LA-UR-05-7985, Los Alamos National Laboratory, 2005.

[2] S. Li and W. J. Rider, Code verification and assessment for adaptive mesh refinement simulations, Technical Report LA-UR-06-4595, Los Alamos National Laboratory, 2006.

[3] A recipe to detect the error in discretization schemes, *Int. J. Numer. Meth. Engng.*, 59 (2004), No.15, p.2065-2087